

# Monopoles and instantons in SU(2) lattice gauge theory

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We investigate the monopole-instanton correlation in SU(2) lattice gauge theory using a renormalisation group inspired smoothing technique. We look at the properties of monopole clusters and their correlation with instantons. Since the action of the smoothed configurations is dominated by instantons we compare the smoothed Monte Carlo lattices to artificially reconstructed configurations with the same instanton content but no other fluctuations. Both parallel and randomly rotated (in group space) instanton ensembles are considered.

## 1. INTRODUCTION

In SU(2) and SU(3) lattice gauge theories a strong correlation has been found between Abelian monopoles and instantons [1]. In the earlier works either prepared instanton configurations or cooled Monte Carlo configurations have been used to demonstrate the connection between monopoles and instantons. Lately renormalization group inspired methods were also applied [2].

As monopoles are generally believed to be responsible for the non-Abelian confinement mechanism, their correlation with instantons led to the assumption that instantons might also play a role in confinement. In the present work we investigate the monopole-instanton correlation in SU(2) lattice gauge theory using a renormalisation group inspired smoothing technique that was originally introduced to study instantons [3]. The smoothing (which consists of an inverse blocking and a subsequent blocking) considerably reduces the noise without affecting the long distance physics. We identify monopoles and instantons on the lattice and look at the properties of monopole clusters and their correlation with instantons. Different configurations with different confining properties are used for this purpose. In a further work we also consider the monopole contribution to the string tension.

Simulations have been performed on an  $8^3 \times 16$

lattice using the FP action of Ref. [3]. 30 configurations have been constructed via Monte Carlo method and smoothed up to nine times. It has been shown in Ref. [3] that the SU(2) string tension does not change during smoothing. Therefore, the physical objects which are responsible for confinement (if there are such objects) should be present on all of these configurations.

The topological charge density has been determined on all configurations. On the 9 times smoothed configurations the size and position of instantons and/or antiinstantons can be determined precisely. As the action on these 9 times smoothed configurations is dominated by instantons, we compare the smoothed Monte Carlo lattices to artificially reconstructed configurations with the same instanton content but no other fluctuations. Both parallel and randomly oriented (in group space) instanton ensembles are considered. The long distance properties of these artificial configurations are known as they are identical with the configurations used in Ref. [4]. In particular, neither the parallel nor the randomly oriented instantons confine. Abelian monopoles have been identified both in the Maximally Abelian Gauge (MAG) and in the Polyakov Gauge (PG). However, in the PG case not all the results are displayed here, as we believe that the effective Abelian model extracted in this gauge has no consistent physical meaning.

## 2. TOPOLOGICAL CHARGE DENSITY AND MONOPOLES

In order to see the monopole-instanton correlation we calculate the average topological charge density on the monopole current links ( $\langle q^2 \rangle_{mon}$ ) and the average charge on the whole lattice ( $\langle q^2 \rangle_{tot}$ ). If there is a correlation then one should find  $\sqrt{\langle q^2 \rangle_{mon} / \langle q^2 \rangle_{tot}} > 1$ . Our results are shown in the following table.

| configuration | Maximal Abelian Gauge | Polyakov Gauge |
|---------------|-----------------------|----------------|
| original      | 1.0969(5)             | 1.002(2)       |
| 1 smoothing   | 1.91(4)               | 1.13(2)        |
| 2 smoothing   | 2.24(6)               | 1.31(3)        |
| 9 smoothing   | 3.28(15)              | 1.91(8)        |
| random        | 4.25(20)              | 2.04(7)        |
| parallel      | 4.33(20)              | 1.89(10)       |

The correlation is much stronger in MAG than in PG at all stages of smoothing. Not unexpectedly, the correlation increases with smoothing in both cases. We obtain, however, maximal correlation for the artificial instanton configurations. The parallel and randomly oriented instanton ensembles do not differ as far as the correlation is concerned. Note that in PG the correlation is essentially the same for the 9 times smoothed configurations and for the artificial ones. In MAG, in contrast, this is much higher for the prepared configurations than for any Monte Carlo generated ones (up to 9 smoothing, of course). It is interesting that the instanton configurations do not confine, and still, the monopole-instanton correlation is the strongest for these, non-confining systems.

## 3. MONOPOLE LOOPS AND INSTANTONS

Quantities characterizing the monopole configurations are shown in an additional table. We observe that the first smoothing step removes roughly 90% of the monopoles. Still, the SU(2) string tension does not change. We have also shown that the Abelian dominance found for the original, untouched configurations also weakens after one smoothing: the Abelian string tension

drops to 70% [5]. Thus, if monopoles alone would provide the full string tension of the effective Abelian model, then in MAG, 10% of monopoles should be responsible for most of the string tension. This is in a qualitative agreement with the results of [6].

| MAG config. | Number of monopoles | Number of loops | Largest loop length |
|-------------|---------------------|-----------------|---------------------|
| original    | 1972(119)           | 77(10)          | 1447(228)           |
| 1 smooth    | 219(36)             | 5.5(2.0)        | 122(40)             |
| 2 smooth    | 165(35)             | 4.9(2.0)        | 95(46)              |
| 9 smooth    | 83(27)              | 4.3(1.5)        | 41(21)              |
| random      | 43(18)              | 5.6(2.0)        | 15(9)               |
| parallel    | 41(16)              | 6.1(2.1)        | 11(7)               |

The average instanton number for the configurations is  $6.7 \pm 2.1$ . The average number of monopole loops for the smoothed configurations as well as for the instanton ensembles, is approximately the same. Is there some explanation behind or this is simply an accident? We studied the number of monopole loops versus the instanton number for all configurations. Correlation analysis of mathematical statistics clearly showed a strong linear correlation between these quantities in case of the artificial instanton configurations. However, no correlation exists between the same quantities for the smoothed Monte Carlo configurations.

This indicates the following. On the artificial instanton configurations (both with parallel and random instanton orientation), in average, each instanton is accompanied by a monopole loop close to its centre. The average instanton radius is  $1.8 \pm 0.6$  and the instanton distribution is dilute (see the average instanton number). The average length of monopole loops is  $6.8 \pm 3.7$  for the parallel and  $7.7 \pm 5.3$  for the randomly oriented instanton ensembles, i.e. the loops are small, local loops (see also the length of the largest loops in the table). In contrast, for the smoothed configurations the average length of the monopole loops is  $39.7 \pm 44.8$  and  $19.7 \pm 17.5$  for the once smoothed and for the 9 times smoothed configurations, respectively. On these configurations, large loops are formed which can cross several instantons and the number of loops and number of instantons is uncorrelated. We illustrate the situ-

ation for a configuration sample in Fig. 1 (which is available in colour from the hep-lat archives). On the real configurations large loops are present which cannot be paired with individual instantons.

#### 4. CONCLUSIONS

Using renormalization group inspired smoothing in SU(2) lattice gauge theory we showed, that the first smoothing step drastically reduces the number of Abelian monopoles. Additional smoothing steps result only in moderate effect.

The correlation between monopoles and instantons increases with smoothing. We find a much stronger correlation for MAG than for PG.

In case of the artificially prepared instanton configurations the size of the monopole loops is determined mainly by the size of the instantons. The monopole-instanton correlation is the strongest for these ensembles. In average, each instanton is accompanied by a small monopole loop. The Monte Carlo generated smoothed configurations differ considerably from the artificial ones. Large loops of monopoles are present. Work is in progress to determine the monopole contribution to the string tension at different stages of the smoothing sequence.

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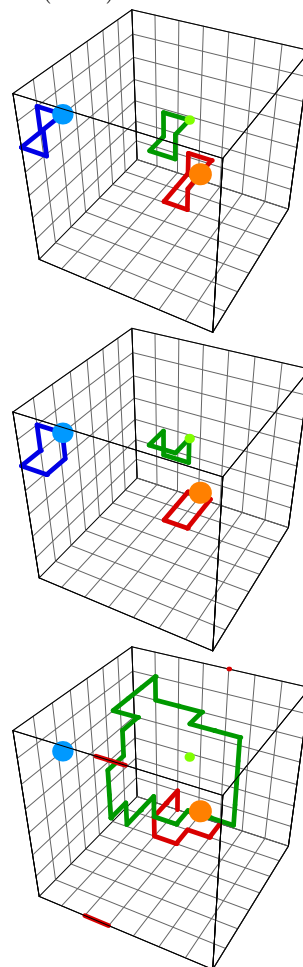


Figure 1. Time projection of a configuration with three instantons for artificial parallel (top) and random (middle) orientation compared to the smoothed real one (bottom). The instanton radii are  $1.5a$ ,  $3.25a$  and  $3.5a$